What Students Need to Learn in Trigonometry

Following is a list of facts that students in all sections of trigonometry should learn. During tests students should not be allowed to use notes or cards listing these, but should recall them or be able to derive them.

Definitions of the trigonometric functions

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x},\ x \neq 0$

$\csc\theta = \frac{r}{y},\ y \neq 0$ $\sec\theta = \frac{r}{x},\ x \neq 0$ $\cot\theta = \frac{x}{y},\ y \neq 0$

Special Angle Values and Quadrantal Angle Values

The student should be able to find, without using a calculator, numerical values of the six trigonometric functions of any angle whose reference angle is a special angle $\frac{\pi}{6}, \frac{\pi}{4}$, or $\frac{\pi}{3}$, whether given in radians or degrees or any quadrantal angle $0, \frac{\pi}{2}, \pi$ or $\frac{3\pi}{2}$ or any angle coterminal with a quadrantal angle.

The Domains and Ranges of the Circular Functions: In the table below, *n* = an integer

	Domain	Range
Sin s	$(-\infty, \infty)$	$[-1, 1]$
Cos s	$(-\infty, \infty)$	$[-1, 1]$
Tan s	$s \neq \frac{\pi}{2} + n\pi$	$(-\infty, \infty)$
Sec s	$s \neq \frac{\pi}{2} + n\pi$	$(-\infty, -1] \cup [1, \infty)$
Csc s	$s \neq n\pi$	$(-\infty, -1] \cup [1, \infty)$
Cot s	$s \neq n\pi$	$(-\infty, \infty)$

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$

$\cot\theta = \frac{1}{\tan\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Pythagorean Identities

$\cos^2\theta + \sin^2\theta = 1$ $\qquad 1 + \tan^2\theta = \sec^2\theta$ $\qquad \cot^2\theta + 1 = \csc^2\theta$

Even-Odd Identities

$\sin(-\theta) = -\sin\theta$ $\qquad \csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$ $\qquad \sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\qquad \cot(-\theta) = -\cot\theta$

Sum and Difference Identities for Sine, Cosine, and Tangent

$$\cos(A \mp B) = \cos A \cos B \pm \sin A \sin B$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double Identities for Sine, Cosine, and Tangent

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $\qquad \cos(2\theta) = 2\cos^2\theta - 1$ $\qquad \cos(2\theta) = 1 - 2\sin^2\theta$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half Angle Identities for Sine, Cosine, and Tangent

$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$ $\qquad \sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ $\qquad \tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta}$ $\qquad \tan\left(\frac{\theta}{2}\right) = \frac{\sin\theta}{1+\cos\theta}$

Arc Length and Area of a Circle Sector

$s = r\theta$, θ in radians $\qquad A = \frac{1}{2}r^2\theta$, θ in radians

Angular and Linear Speed

$\omega = \frac{\theta}{t}$, θ in radians $\qquad v = \frac{s}{t}$

One period of the graph of the six trigonometric functions

The Domains and Ranges of the Inverse Trigonometric Functions (Sine, Cosine, and Tangent only):

Function	**Domain**	**Range**
$y = \arcsin x$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$y = \arccos x$	$[-1,1]$	$[0,\pi]$
$y = \arctan x$	$(-\infty,\infty)$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

The Law of Sines and the Law of Cosines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad\qquad c^2 = a^2 + b^2 - 2ab\cos C$$

The dot product formula and the geometric interpretation of the dot product

$$u \bullet v = u_1 \cdot v_1 + u_2 \cdot v_2 \qquad\qquad \cos\theta = \frac{u \bullet v}{|u||v|}$$

Polar Coordinates

Translation equations: $x = r\cos\theta$, $y = r\sin\theta$, $x^2 + y^2 = r^2$, $\tan\theta = \frac{y}{x}$, $x \neq 0$

Equation of a line passing through the pole: $\theta = c$, c a real number

Equation of a circle of radius a centered at the pole: $r = a$, a a real number

Parametric Equations

The Unit Circle: $x(t) = \cos t$, $y(t) = \sin t$ $\quad 0 \le t \le 2\pi$